interval insertion
interval intersection search

click to begin demo
Interval search trees

interval search tree

binary search tree (left endpoint is key)

max endpoint in subtree rooted at node
interval insertion
interval intersection search
Inserting an interval

To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval \((16, 22)\)**

```
(7, 10) 10
(15, 18) 18
(5, 8) 18
(17, 19) 24
(4, 8) 8
(21, 24) 24
```
To insert an interval \( (lo, hi) \):

- Insert into BST, using \( lo \) as the key.
- Update max in each node on search path.

**insert interval \( (16, 22) \)**
Inserting an interval

To insert an interval \((lo, hi)\):

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**insert interval \((16, 22)\)**
To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval (16, 22)**

```plaintext
• Insert into BST, using 16 as the key.
• Update max in each node on search path.
```
Inserting an interval

To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval \((16, 22)\)**

```plaintext
Inserting an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval \((16, 22)\)**

```

```plaintext
(5, 8)
(4, 8) 8
(7, 10) 10
(15, 18) 18
(17, 19) 24
(21, 24) 24
(16, 22)
```
To insert an interval \((lo, hi)\):
- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

Inserting an interval \((16, 22)\)
Inserting an interval

To insert an interval \((lo, hi)\):
- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

Inserting an interval

\[\begin{align*}
(17, 19) \\
(5, 8) \\
(4, 8) \\
(7, 10) \\
(15, 18) \\
(16, 22) \\
(15, 18) \\
(21, 24)
\end{align*}\]
Inserting an interval

To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

insert interval \((16, 22)\)

```
(17, 19) 24

(5, 8) 18

(4, 8) 8

(15, 18) 18

(7, 10) 10

(16, 22) no more tree (insert here)
```
Inserting an interval

To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval \((16, 22)\)**
Inserting an interval

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insert interval \((16, 22)\)
Inserting an interval

To insert an interval \((lo, hi)\):

- Insert into BST, using \(lo\) as the key.
- Update max in each node on search path.

**insert interval \((16, 22)\)**
› interval insertion
› interval intersection search
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):
• If interval in node intersects query interval, return it.
• If left subtree is null, go right.
• If max endpoint in left subtree is less than \(lo\), go right.
• Else go left.

interval intersection
search for \((23, 25)\)
To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection search for \((23, 25)\)**

- Compare \((23, 25)\) to \((17, 19)\). (no intersection)

**Tree Diagram**

- Root: \((17, 19)\) with max endpoint 24
- Left child: \((5, 8)\) with max endpoint 22
  - Left child: \((4, 8)\) with max endpoint 8
  - Right child: \((15, 18)\) with max endpoint 22
- Right child: \((21, 24)\) with max endpoint 24
- Right child of \((21, 24)\): \((16, 22)\) with max endpoint 22
  - Right child: \((10, 16)\)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):
- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**Interval intersection search for** \((23, 25)\)

```
(17, 19)---24
\|
(5, 8)---22
\|
(4, 8)---8
\|            \|            \|
\|            \|            \|
(7, 10)---10 (15, 18)---22 (21, 24)---24
```

compare 22 to 23  
(no intersection in left, go right)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

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- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

Interval intersection

Search for \((23, 25)\)

\[(4, 8) \rightarrow (5, 8) \rightarrow (17, 19) \rightarrow (23, 25) \rightarrow (17, 19) \rightarrow (16, 22) \rightarrow (15, 18) \rightarrow (5, 8)\]
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
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- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**Interval intersection search for \((23, 25)\)**

[Diagram showing an example of searching for an interval intersection]
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

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- Else go left.

**interval intersection**

search for \((12, 14)\)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection search for \((12, 14)\)**

- Compare \((12, 14)\) to \((17, 19)\): (no intersection)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection search for \((12, 14)\)**

```
(12, 14)  
(17, 19)  
(5, 8)    
(4, 8)    
(15, 18)  
(7, 10)   
(16, 22)  
(21, 24)  
```

compare 22 to 12 (go left)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
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- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

interval intersection search for \((12, 14)\)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
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- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection search for (12, 14)**

compare \((12, 14)\) to \((5, 8)\)
(no intersection)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

• If interval in node intersects query interval, return it.
• If left subtree is null, go right.
• If max endpoint in left subtree is less than \(lo\), go right.
• Else go left.

**interval intersection search for (12, 14)**
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):
- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

interval intersection
search for \((12, 14)\)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection search for \((12, 14)\)**

```plaintext
compare (12, 14) to (15, 18)
(no intersection)
```
To search for an interval that intersects query interval \((lo, hi)\):

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

```
interval intersection
search for (12, 14)
```

```tree
(5, 8)  
   (4, 8)  (12, 14)  (15, 18)  (21, 24)  
   (7, 10) (16, 22)  
```

compare 10 to 12 (go right)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):
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- Else go left.

interval intersection
search for \((12, 14)\)
Search for an intersecting interval

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**interval intersection search for \((12, 14)\)**
Search for an intersecting interval

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```
interval intersection
search for (12, 14)
```

![Diagram showing interval intersection search]

- Compare (12, 14) to (16, 22) (no intersection)
Search for an intersecting interval

To search for an interval that intersects query interval $(lo, hi)$:

- If interval in node intersects query interval, return it.
- If left subtree is null, go right.
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- Else go left.

**Interval intersection search for $(12, 14)$**
Search for an intersecting interval

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interval intersection
search for \((21, 23)\)
Search for an intersecting interval

To search for an interval that intersects query interval \((lo, hi)\):

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- If left subtree is null, go right.
- If max endpoint in left subtree is less than \(lo\), go right.
- Else go left.

**interval intersection**
**search for (21, 23)**

- **(4, 8)**
  - **(5, 8)**
    - **(15, 18)**
      - **(7, 10)**
      - **(16, 22)**
    - **(22, 24)**
  - **(21, 24)**

**compare (21, 23) to (17, 19)**
(no intersection)
Search for an intersecting interval

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interval intersection
search for \((21, 23)\)
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interval intersection
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**interval intersection**

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**interval intersection**

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To search for an interval that intersects query interval \((lo, hi)\):

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- Else go left.

interval intersection search for \((21, 23)\)

compare \((21, 23)\) to \((16, 22)\) (intersection!)