Lecture 4

More on Data Flow:
Constant Propagation
Control Flow: Speed, Loops

I Constant Propagation

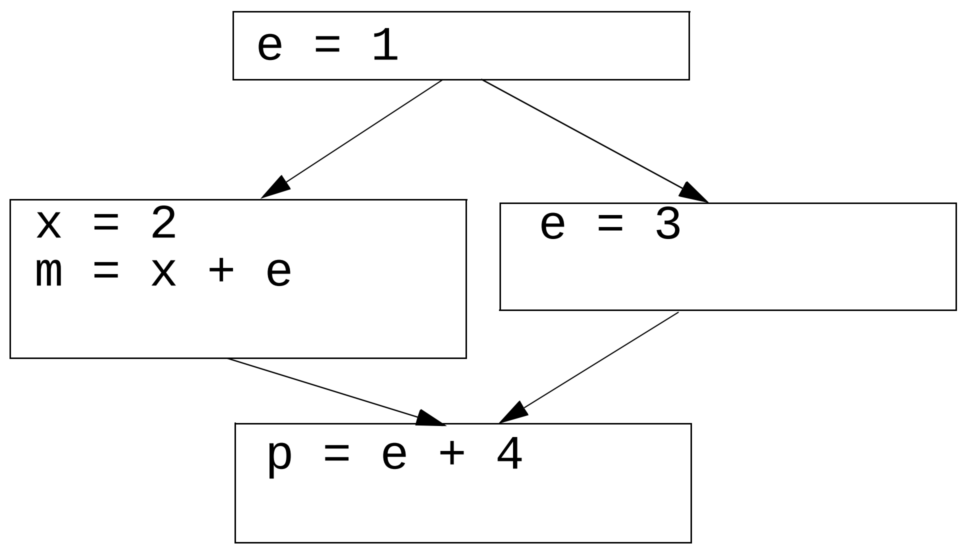
II Efficiency of Data Flow Analysis

III Algorithm to find loops

Reading: Chapter 9.4, 9.6

I. Constant Propagation/Folding

- **At every basic block boundary, for each variable v**
  - determine if v is a constant
  - if so, what is the value?



Semi-lattice Diagram

- Finite domain?
- Finite height?

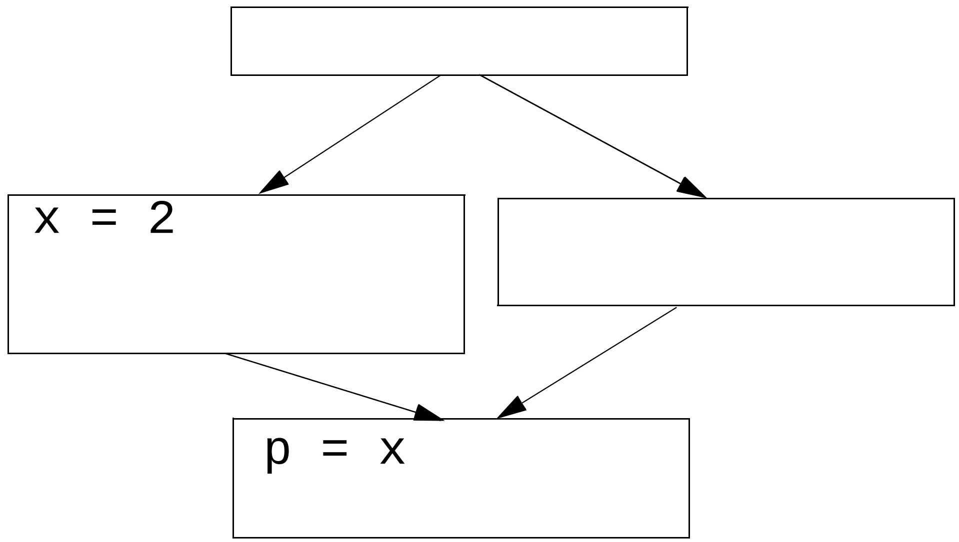
Equivalent Definition

- **Meet Operation:**

v1	v2	v1 $\wedge$ v2
undef	undef	undef
	c_2	c_2
	NAC	NAC
c_1	undef	c_1
	c_2	c_1, if $c_1 = c_2$ NAC otherwise
	NAC	NAC
NAC	undef	NAC
	c_2	NAC
	NAC	NAC

- Note: undef $\wedge$ c2 = c2!

Example



Transfer Functions

- **Assume a basic block has only 1 instruction**
- **Let IN[b,x], OUT[b,x]**
  - be the information for variable x at entry and exit of basic block b
- **OUT[entry, x] = undef, for all x.**
- **Non-assignment instructions: OUT[b,x] = IN[b,x]**
- **Assignment instructions: (next page)**

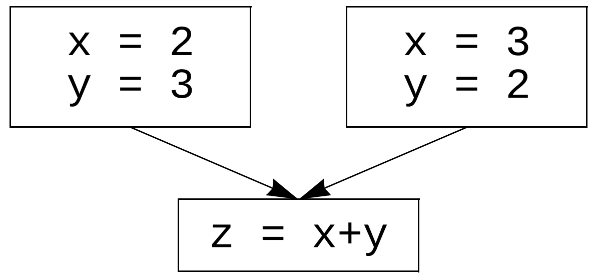
Constant Propagation (Cont.)

- **Let an assignment be of the form $x_3 = x_1 + x_2$**
  - + represents a generic operator
  - $OUT[b,x] = IN[b,x]$, if $x \neq x_3$

$IN[b,x_1]$	$IN[b,x_2]$	$OUT[b,x_3]$
undef	undef	
	c_2	
	NAC	
c_1	undef	
	c_2	
	NAC	
NAC	undef	
	c_2	
	NAC	

- **Use: $x \le y$ implies $f(x) \le f(y)$ to check if framework is monotone**
  - $[v_1\ v_2\ \dots] \le [v_1'\ v_2'\ \dots]$, $f([v_1\ v_2\ \dots]) \le f([v_1\ v_2\ \dots])$

Distributive?

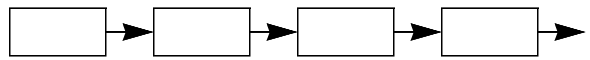


Summary of Constant Propagation

- **A useful optimization**
- **Illustrates**
    - abstract execution
    - an infinite semi-lattice
    - a non-distributive problem

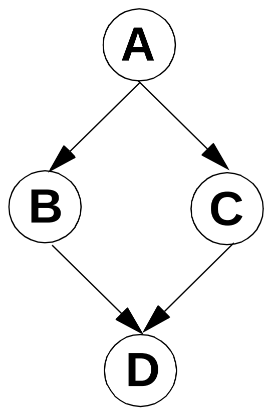
II. Efficiency of Iterative Data Flow

- **Assume forward data flow for this discussion**
- **Speed of convergence depends on the ordering of nodes**

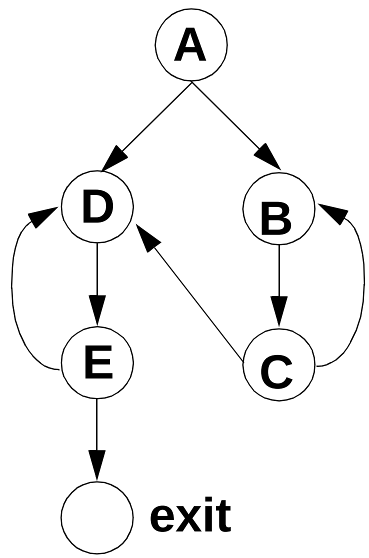


- **How about:**

I.



II.



Depth-first Ordering: Reverse Postorder

- **Preorder traversal: visit the parent before its children**
- **Postorder traversal: visit the children then the parent**
- **Preferred ordering: reverse postorder**
- **Intuitively**
  - depth first postorder visits the farthest node as early as possible
  - reverse postorder delays visiting farthest node

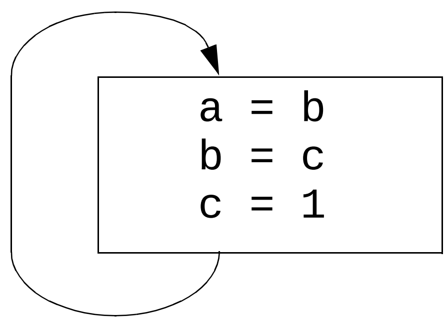
“Reverse Post-Order” Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
  OUT[Entry] = ∅

// Initialization for iterative algorithm
  For each basic block B other than Entry
      OUT[B] = ∅

// iterate
  While (changes to any OUT occur) {
     For each basic block B other than Entry
        in reverse post order {
        in[B] = ∪ (out[p]), for all predecessors p of B
        out[B] = fB(in[B]) // out[B]=gen[B]∪(in[B]-kill[B])
     }
```

Consideration in Speed of Convergence

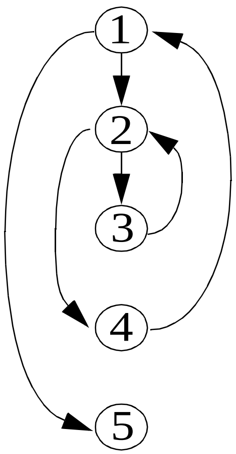
Does it matter if we go around the same cycle multiple times?

- **Cycles do not make a difference:**
  - reaching definitions, liveness
- **Cycles make a difference:** constant propagation



Speed of Convergence

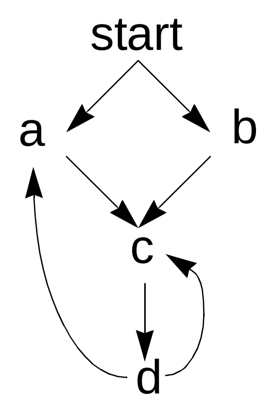
- If cycles do not add info:
  - Labeling nodes in a path by their reverse postorder rank:
    1 -> 4 -> 5 -> 7 -> 2 -> 4 ...
  - info flows down nodes of increasing reverse postorder rank in 1 pass
- Loop depth = max. # of “retreating edges” in any acyclic path
- **Maximum** # iterations in data flow algorithm = Loop depth+2
  (2 is necessary even if there are no cycles)



- Knuth’s experiments show: average loop depth in real programs = 2.75

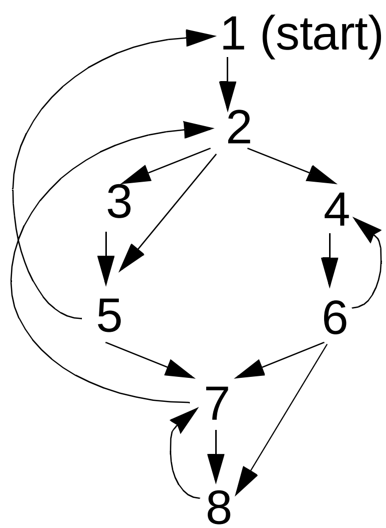
III. What is a Loop?

- **Goal:**
  - Define a loop in graph-theoretic terms (control flow graph)
  - Not sensitive to input syntax,
    a uniform treatment for all loops: DO, while, goto’s
- **Informally: A “natural” loop has**
  - edges that form at least a cycle
  - a single entry point



Dominators

- Node d dominates node n in a graph (d *dom* n) if every path from the start node to n goes through d
  - a node dominates itself



- Immediate dominance:
  d idom n : d *dom* n, $d \neq n$, $\neg\exists m$ s.t. d *dom* m and m *dom* n
- Immediate dominance relationships form a tree

Finding Dominators

- **Definition**
  - Node d dominates node n in a graph (d dom n) if every path from the start node to n goes through d
- **Formulated as a MOP problem**
  - node d lies on all possible paths reaching node $n \Rightarrow d\ dom\ n$
    - Direction:
    - Values:
    - Meet operator:
    - Top:
    - Bottom:
    - Boundary condition: start/exit node =
    - Finite descending chains only?
    - Transfer function:
- **Speed:**
    - With reverse postorder, solution to most flow graphs (reducible flow graphs) found in 1 pass

Definition of Natural Loops

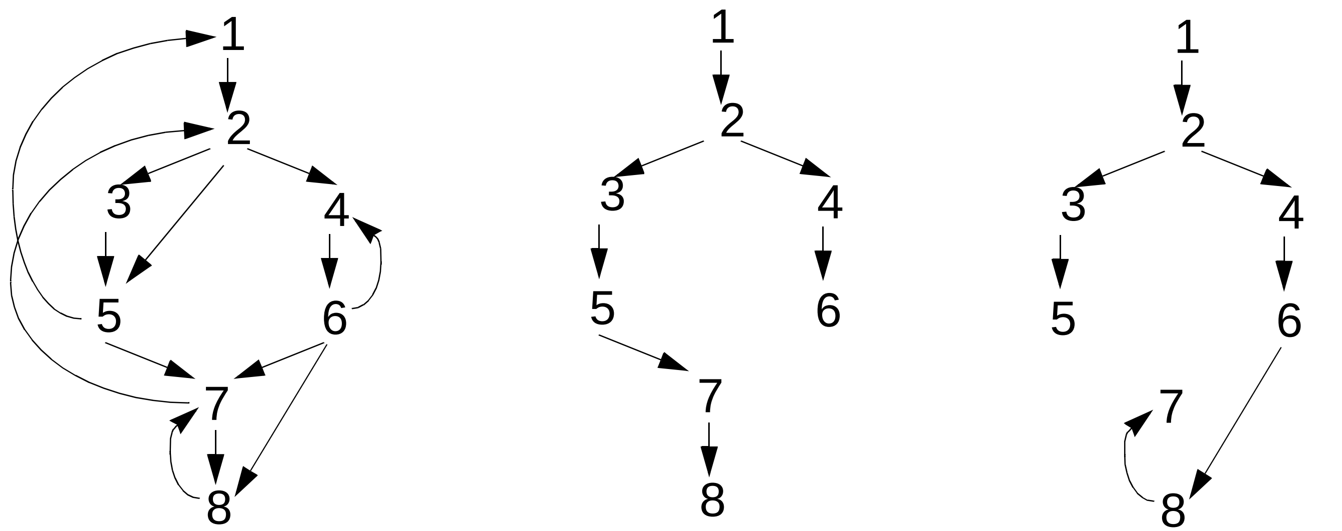
- Single entry-point: ***header*** (d)
  a header dominates all nodes in the loop

- A ***back edge*** ($n \rightarrow d$) in a flow graph is
  an edge whose destination dominates its source (d dom n)

- The ***natural loop of a back edge*** ($n \rightarrow d$) is
  d + { nodes that can reach n without going through d }.

Graph Edges

- **Depth-first spanning tree**
  - Edges traversed in a depth-first search of a graph form a depth-first spanning tree



- **Categorizing edges in graph**
  - Advancing edges: from ancestor to proper descendant
  - Retreating edges: from descendant to ancestor
    (not necessarily proper)
  - Cross edges: all other edges

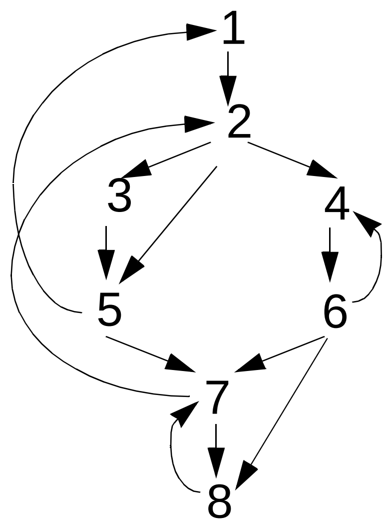
Back Edges

- **Definition**
  - Back edge: $n \rightarrow d$, d dom n
- **Relationships between graph edges and back edges**
  - a back edge must be a retreating edge
    dominator $\Rightarrow$ visit d before n, n must be a descendant of d
  - a retreating edge is not necessarily a back edge
- **Most programs (all structured code, and most GOTO programs)**
  - retreating edges = back edges

Constructing Natural Loops

- The ***natural loop of a back edge*** ($n \rightarrow d$) is
  d + { nodes that can reach n without going through d }.

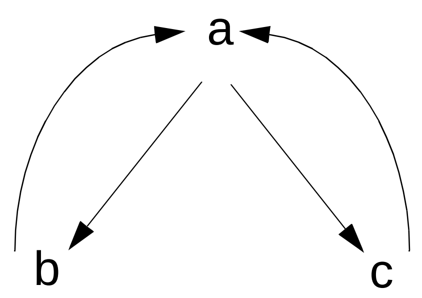
- Remove d from the flow graph, find all predecessors of n
- Example



Inner Loops

- **If two loops do not have the same header**
  - they are either disjoint, or
  - one is entirely contained (nested within) the other
    -- inner loop, one that contains no other loop.

- **If two loops share the same header**
  - Hard to tell which is the inner loop
  - Combine as one



Summary

- **Constant propagation**
- **Introduced the reverse postorder iterative algorithm**
- **Define loops in graph theoretic terms**
- **Definitions and algorithms for**
  - Dominators
  - Back edges
  - Natural loops